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Erratum

Gust Energy Extraction for Mini and Micro Uninhabited Aerial Vehicles

Jack W. Langelaan
Pennsylvania State University, University Park, Pennsylvania 16802
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In the derivation of rate of change of specific total energy an error was made in the derivation from Eq. (22) to Eq. (23). The correct expression is

$$\dot{E}_{\text{tot}} = \frac{qS}{m}[(-C_D + C_T \cos\alpha)(v_a + w_{i,x}\cos\gamma - w_{i,z}\sin\gamma) - (C_L + C_T\sin\alpha)(w_{i,x}\sin\gamma + w_{i,z}\cos\gamma)] \tag{1}$$

Wind gradients do not appear explicitly in the expression for total power expressed with respect to the inertial frame. Note however that time- and spatially-varying winds will result in time varying forces (through changes in airspeed and angle of attack), so wind gradients will indirectly affect total power.

However, expressing total energy using airspeed for the kinetic energy term gives

$$\dot{E}_{\text{air}} = g\dot{h} + \dot{v}_a v_a \tag{2}$$

Substituting dynamics gives

$$\dot{E}_{\text{air}} = g\dot{h} + \left[\frac{qS}{m}(-C_D + C_T\cos\alpha)v_a - \dot{w}_{i,x}v_a\cos\gamma + \dot{w}_{i,z}v_a\sin\gamma - v_a g\sin\gamma\right] \tag{3}$$

Recognizing that $\dot{h} = -\dot{z} = v_a\sin\gamma - w_{i,z}$,

$$\dot{E}_{\text{air}} = -g w_{i,z} + \left[\frac{qS}{m}(-C_D + C_T\cos\alpha)v_a - \dot{w}_{i,x}v_a\cos\gamma + \dot{w}_{i,z}v_a\sin\gamma\right] \tag{4}$$

Assuming a frozen gust field,

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{w} = \nabla\mathbf{w}\begin{bmatrix}\dot{x}_i\\ \dot{z}_i\end{bmatrix} = \begin{bmatrix}\frac{\delta w_x}{\delta x_i}\dot{x}_i + \frac{\delta w_x}{\delta z_i}\dot{z}_i\\ \frac{\delta w_z}{\delta x_i}\dot{x}_i + \frac{\delta w_z}{\delta z_i}\dot{z}_i\end{bmatrix} \tag{5}$$

where $\nabla\mathbf{w}$ is the spatial gradient of the wind vector. Equivalently,

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{w} = \nabla\mathbf{w}\begin{bmatrix}v_a\cos\gamma + w_{i,x}\\ -v_a\sin\gamma + w_{i,z}\end{bmatrix} = \nabla\mathbf{w}\begin{bmatrix}v_a\cos\gamma\\ -v_a\sin\gamma\end{bmatrix} + \nabla\mathbf{w}\begin{bmatrix}w_{i,x}\\ w_{i,z}\end{bmatrix} \tag{6}$$

The rate of change of total energy is therefore

$$\dot{E}_{\text{air}} = -g w_{i,z} + \frac{qS}{m}(-C_D + C_T\cos\alpha)v_a - \mathbf{v}_a^T[\nabla\mathbf{w}]\mathbf{v}_a - \mathbf{v}_a^T[\nabla\mathbf{w}]\mathbf{w} \tag{7}$$

where $\mathbf{v}_a = [v_a\cos\gamma - v_a\sin\gamma]^T$.

The influence of wind gradients on airmass-relative total power is contained in the third and fourth terms of the right-hand side of Eq. (7). The $\mathbf{v}_a^T[\nabla\mathbf{w}]\mathbf{v}_a$ term is likely to be larger in magnitude since airspeed is typically larger than wind speed. It will contribute to energy gain under two conditions: first, when $\nabla\mathbf{w}$ is negative and γ is negative; second, when $\nabla\mathbf{w}$ is positive and γ is positive. Note that for steady-state gliding flight γ is always negative.

Using Eq. (7) above to compute optimal speeds to fly gives results that differ from those in the original work an average of 0.06%, with a maximum speed error of 1.8%. The largest absolute difference in rate of change of energy is 0.024 (a difference of less than 1%), which occurs when airspeed is low and wind gradient is large. This error has a negligible effect on the remainder of the original paper's results.